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Original Article

A study on hereditary generalized regular submaximal spaces

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Abstract

In this paper the concepts of dense-H-sets, codense-H-sets, hereditary generalized submaximal spaces and hereditary generalized regular submaximal spaces are introduced with necessary examples. Also interesting properties and some equivalent statements of hereditary generalized regular submaximal spaces are discussed.

Keywords: Dense-$\mathcal{H}$-sets, Hereditary generalized submaximal space, g-$\mathcal{H}$-open, Pre regular-$\mathcal{H}$-open set, Regular-$\mathcal{H}$-closed

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1. Introduction

The concept of submaximality of general topological spaces was introduced by Hewitt [13] in 1943. He discovered a general way of constructing maximal topologies. In [3], Alas et al. proved that there can be no dense maximal subspace in a product of first countable spaces, while under Booth's Lemma there exists a dense submaximal subspace in $[0,1]^c$. The first systematic study of submaximal spaces was undertaken in the paper of Arhangel'ski,[3] and Collins [4]. In this paper, several characterizations and further properties of regular $\mathcal{H}$-submaximal in hereditary generalized topological spaces are obtained.

The class of submaximal spaces (as well as the name for it) was introduced by Bourbaki [7]. One of the reasons to consider submaximal spaces is provided by the theory of maximal spaces. A space X is called maximal if it is dense-in-itself and no larger topology on the set X is dense-in-itself. It was shown [17, 18], that a space is maximal if and only if it is an extremally disconnected submaximal space without isolated points. Secondly, any connected Hausdorff space which does not admit a larger connected topology is submaximal [12]. Third, submaximal spaces were characterized by Bourbaki as spaces that do not admit a larger topology with the same semi-regularisation [7, p. 1391]. Fourth, nonempty maximal spaces are not decomposable into two nonempty dense complementary subspaces just because they are submaximal-obviously, dense open subsets cannot be disjoint!

2. Preliminaries

Definition 1.2.1 [9]

Let X be a nonempty set and let expX be the power of X. The collection μ of subset of X satisfying the following conditions is called generalized topology,

(a) $\emptyset \in \mu$;

(b) $G_i \in \mu$ for $i \in I$ implies $G = \bigcup_{i\in I} G_i \in \mu$.

The elements of μ are called μ-open and their compliments are called μ-closed. The pair (X,μ) is called a generalized topological spaces (GTS).

Definition 1.2.2 [9]

Let (X, μ) be a generalized topological space. Let A be any subset of X. Then the interior of A is defined as the union of all μ-open sets contained in A and it is denoted by $i_\mu(A)$. That is,

$i_\mu(A) = \cup \{ A \subset X : U \subset A \text{ and } U \in \mu \}$.

If A is open, then $A = i_\mu(A)$.

Definition 1.2.3 [9]

Let (X, μ) be a generalized topological space. Let A be any subset of X. Then the closure of A is defined as the intersection of all closed sets containing A and it is denoted by cl(A) or $\overline{A}$. That is,

$$c_\mu(A) = \cap \{ A \subset X : U \subset A, U^C \in \mu \}.$$

If A is closed, then $A = c_\mu(A)$.

Definition 1.2.4 [9]

Let X be a nonempty set. A hereditary class $\mathcal{H}$ of X if $A \in \mathcal{H}$ and $B \subset A$ then $B \in \mathcal{H}$. A generalized topological spaces (X, μ) with a hereditary class $\mathcal{H}$ is Hereditary Generalized Topological Spaces (HGTS) and denoted by (X, μ, $\mathcal{H}$).

For each $A \subset X$, $A^*(\mathcal{H}, \mu) = \{ x \in X : A \cap G \notin \mathcal{H}$ for every $G_i \in \mu$ such that $x \in G \}$. If there is no ambiguity then we write A* in place of $A^*(\mathcal{H}, \mu)$. For each $A \subset X$, then $c^*_\mu(A) = A \cup A^*$.

Definition 1.2.5 [15]

Any subset A of a topological spaces (X, μ) is said to be dense in X if cl(A) = X.

Definition 1.2.6 [16]

Any topological space (X, μ) is said to be a submaximal space if every dense subset of X is open.

Definition 1.2.7 [22]

Any topological space (X, μ) is said to be a regular submaximal space if every dense set in (X, μ) is regular open.

Definition 1.2.8 [6]

A topological space (X, μ) is said to be a g-submaximal space if every dense set is g-open.

Definition 1.2.9 [8]

Let X and Y be topological spaces. A function $f : X \to Y$ is called a closed map is f(F) is closed in Y whenever F is closed in X.

Definition 1.2.10 [8]

Let X and Y be topological spaces. A function $f : X \to Y$ is called a open map is f(F) is open in Y whenever F is open in X.

3. Hereditary generalized regular submaximal space

Definition 2.1

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be regular-$\mathcal{H}$-open if $A = i_\mu(c^*_\mu(A))$. The complement of a regular-$\mathcal{H}$-open set is said to be a regular-$\mathcal{H}$-closed.

Example 2.1

Let X = { a, b, c, d, e }, μ = { ϕ, { a }, { c }, { a, c }, { a, b, c }, { c, d }, { a, c, d }, { a, b, c, d } } and $\mathcal{H}$ = { ϕ, { a }, { b }, { c } }. Clearly, μ is a generalized topology and $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Let A = { a, b, c, d } be a subset of X. Then, A* = { a, b, c, d } and so $c^*_\mu(A) = A \cup A^*$ = { a, b, c, d }. Then, $i_\mu(c_\mu^*(A)) = \{$ a, b, c, d $\}$. Hence, A = $i_\mu(c_\mu^*(A))$. Therefore, A is regular-$\mathcal{H}$-open.

Definition 2.2

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space and A be any subset of X. Then the regular $\mathcal{H}$-interior of A (briefly, $R_{\mathcal{H}}$int(A)) is defined by

$R_{\mathcal{H}}$int(A) = ∪ { G ; G ⊆ A and each G ⊆ X is a regular $\mathcal{H}$-open set }.

Definition 2.3

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space and A be any subset of X. Then the regular $\mathcal{H}$-closure of A (briefly, $R_{\mathcal{H}}$cl(A)) is defined by

$R_{\mathcal{H}}$cl(A) = ∩ { K ; A ⊆ K and each K ⊆ X is a regular $\mathcal{H}$-closed set }.

Definition 2.4

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be dense-$\mathcal{H}$-set if $c_\mu^*(A) = X$.

Definition 2.5

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be codense-$\mathcal{H}$-set if X\A is dense-$\mathcal{H}$-set.

Example 2.2

Let X = { a, b, c }, μ = { ϕ, X, { a }, { a, b }, { b, c } } and $\mathcal{H}$ = { ϕ, { a } }. Clearly, μ is a generalized topology, $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Let A = { b, c } be a subset of X. Then A^* = { a, b, c }. Hence $c_\mu^*(A)$ = X. Therefore, A is dense $\mathcal{H}$ set. Hence X\A = { a } is codense-$\mathcal{H}$-set.

Definition 2.6

Any hereditary generalized topological space (X, μ, $\mathcal{H}$) is said to be a hereditary generalized submaximal space if every dense-$\mathcal{H}$-set (resp. codense-$\mathcal{H}$-set) in (X, μ, $\mathcal{H}$) is μ-open (resp. μ-closed).

Example 2.3

Let X = { a, b, c }, μ = { ϕ, { c }, { a, c }, { b, c }, X } and $\mathcal{H}$ = { ϕ, { a }, { b }, { a, b } }. Clearly, μ is a generalized topology, $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Then dense-$\mathcal{H}$-sets are { c }, { a, c }, { b, c } and X. Clearly all dense-$\mathcal{H}$-sets are μ-open sets. Hence (X, μ, $\mathcal{H}$) is said to be hereditary generalized submaximal space.

Definition 2.7

Any hereditary generalized topological space (X, μ, $\mathcal{H}$) is said to be a hereditary generalized regular submaximal space if every dense-$\mathcal{H}$-set in (X, μ, $\mathcal{H}$) is regular-$\mathcal{H}$-open.

Example 2.4

Let X = { a, b, c }, μ = { ϕ, { c }, { a, c }, { b, c }, X } and $\mathcal{H}$ = { ϕ, { a }, { b }, { c }, { a, b }, { a, c }, { b, c } }. Clearly, μ is a generalized topology, $\mathcal{H}$ is a hereditary class and the triple (X, μ, $\mathcal{H}$) is a hereditary generalized topological space. Then dense-$\mathcal{H}$-set is X. Clearly X is μ-open set. Hence (X, μ, $\mathcal{H}$) is said to be hereditary generalized regular submaximal space.

Theorem 2.1

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. For any subset A of X, the following statements are equivalent:

(a) X is hereditary generalized regular submaximal,

(b) Every codense-$\mathcal{H}$-subset A of X is regular-$\mathcal{H}$-closed.

Proof:

(a) ⇒ (b)

Let A be a codense-$\mathcal{H}$-subset of X. Since X\A is dense-$\mathcal{H}$-set, by (a), X\A is regular-$\mathcal{H}$-open. Thus, A is regular-$\mathcal{H}$-closed.

(b) ⇒ (a)

Let A be a dense-$\mathcal{H}$-subset of X. Since X\A is codense-$\mathcal{H}$-set, X\A is regular-$\mathcal{H}$-closed. Thus A is regular-$\mathcal{H}$-open.

Proposition 2.1

Every hereditary generalized regular submaximal space is a hereditary generalized submaximal space.

Proof:

Let (X, μ, $\mathcal{H}$) be a hereditary generalized regular submaximal space and A ⊆ X be a dense-$\mathcal{H}$-set. Since (X, μ, $\mathcal{H}$) is hereditary generalized regular submaximal, A is regular-$\mathcal{H}$-open in (X, μ, $\mathcal{H}$). Since every regular-$\mathcal{H}$-open is μ-open, A is μ-open. Hence (X, μ, $\mathcal{H}$) is a hereditary generalized submaximal space as every dense-$\mathcal{H}$-set is a μ-open set in (X, μ, $\mathcal{H}$).

Definition 2.8

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be

g-$\mathcal{H}$-open if $c_\mu^*(A)$ ⊆ U whenever A ⊆ U and U is μ-closed.

Note 2.1

Every regular-$\mathcal{H}$-open is g-$\mathcal{H}$-open.

Proposition 2.2

Every hereditary generalized regular submaximal space is hereditary generalized g-submaximal space.

Proof:

Let A ⊆ X be dense-$\mathcal{H}$-set in a hereditary generalized regular submaximal space (X, μ, $\mathcal{H}$). Since every dense-$\mathcal{H}$-set of X is a regular-$\mathcal{H}$-open, A is regular-$\mathcal{H}$-open in (X, μ, $\mathcal{H}$). Since every regular-$\mathcal{H}$-open is g-$\mathcal{H}$-open, A is g-$\mathcal{H}$-open. Hence every hereditary generalized regular submaximal space is hereditary generalized g-submaximal space.

Definition 2.9

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Any subset A of X is said to be pre regular-$\mathcal{H}$-open if A ⊆ $R_{\mathcal{H}}$int($c_\mu^*(A)$).

Proposition 2.3

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. For any subset A of X, the following statements are equivalent:

(a) A is pre regular-$\mathcal{H}$-open

(b) A = U ∩ D where U is regular-$\mathcal{H}$-open and D is dense-$\mathcal{H}$-set in X.

Proof:

(a) ⇒ (b)

If A is pre regular $\mathcal{H}$-open, then A ⊆ $R_{\mathcal{H}}$int(c_μ^*(A)). Let U = $R_{\mathcal{H}}$int($c_\mu^*(A)$), then U is a regular $\mathcal{H}$-open set. Let D = X − (U − A) = (X − U) ∪ A.

Since $R_{\mathcal{H}}$int($c_\mu^*(A)$) ⊆ $c_\mu^*(A)$ and so $-R_{\mathcal{H}}$int($c_\mu^*(A)$) ⊇ $-\,c_\mu^*(A)$.

Hence X − $R_{\mathcal{H}}$int($c_\mu^*(A)$) ⊇ X − $c_\mu^*(A)$ from which X − U ⊇ X − $c_\mu^*(A)$.

Also, X = $c_\mu^*(A)$ ∪ (X − $c_\mu^*(A)$)

⊆ $c_\mu^*(A)$ ∪ (X − U)

⊆ $c_\mu^*(A)$ ∪ c_μ^*(X − U)

⊆ c_μ^*[A ∪ (X − U)] ⊆ $c_\mu^*(D)$.

Thus X ⊆ $c_\mu^*(D)$. Further $c_\mu^*(D)$ ⊆ X. Therefore X = $c_\mu^*(D)$. Hence D is dense-$\mathcal{H}$-set. Therefore A = U ∩ D.

(b) ⇒ (a)

If A = U ∩ D, where U ∈ regular $\mathcal{H}$-open and D is dense $\mathcal{H}$-set, then A ⊆ U, $R_{\mathcal{H}}$int($c_\mu^*(A)$) ⊆ $R_{\mathcal{H}}$int($c_\mu^*(U)$). Since U = U ∩ X = U ∩ $c_\mu^*(D)$ ⊆ c_μ^*(U ∩D) = $c_\mu^*(A)$. Hence U ⊆ $c_\mu^*(A)$. Then $R_{\mathcal{H}}$int($c_\mu^*(U)$) ⊆ $R_{\mathcal{H}}$int($c_\mu^*(A)$). Therefore, $R_{\mathcal{H}}$int($c_\mu^*(U)$) = $R_{\mathcal{H}}$int($c_\mu^*(A)$). So that A is pre regular $\mathcal{H}$-open.

Proposition 2.4

Let (X, μ, $\mathcal{H}$) be a hereditary generalized topological space. Then the following statements are equivalent:

(a) (X, μ, $\mathcal{H}$) is a hereditary generalized regular submaximal space.

(b) Every pre regular-$\mathcal{H}$-open set is regular-$\mathcal{H}$-open.

Proof:

(a) ⇒ (b)

Suppose that (X, μ, $\mathcal{H}$) is hereditary generalized regular submaximal and A ⊆ X be pre regular-$\mathcal{H}$-open. By Proposition 2.3, A = U ∩ D, where U is regular $\mathcal{H}$-open, D is a dense-$\mathcal{H}$-set. Since (X, μ, $\mathcal{H}$) is hereditary generalized regular submaximal, D is regular-$\mathcal{H}$-open. Since intersection of two regular-$\mathcal{H}$-open sets is a regular-$\mathcal{H}$-open set, A is a regular-$\mathcal{H}$-open set.

(b) ⇒ (a)

Let A be any pre regular $\mathcal{H}$-open. By hypothesis A is regular-$\mathcal{H}$-open. By Proposition 2.3, A = U ∩ D, where U is regular $\mathcal{H}$-open, D is a dense-$\mathcal{H}$-set in (X, μ, $\mathcal{H}$). Since A and U are regular $\mathcal{H}$-open, D must be regular $\mathcal{H}$-open in (X, μ, $\mathcal{H}$). Thus every dense-$\mathcal{H}$-set is a regular-$\mathcal{H}$-open set in (X, μ, $\mathcal{H}$). Hence (X, μ, $\mathcal{H}$) is a hereditary generalized regular submaximal space.

Theorem 2.2

For a hereditary generalized topological space (X, μ, $\mathcal{H}$), the following properties are equivalent:

(a) $(X, \mu, \mathcal{H})$ is a hereditary generalized regular submaximal space.

(b) For all $A \subseteq X$, if $A \backslash R_{\mathcal{H}}int(A) \neq \phi$, then $A \backslash R_{\mathcal{H}}int(c_{\mu}^{*}(A)) \neq \phi$.

Proof:

(a) ⇒ (b)

Let $A \subseteq X$ and $A \backslash R_{\mathcal{H}}int(A) \neq \phi$. Suppose that $A \backslash R_{\mathcal{H}}int(c_{\mu}^{*}(A)) = \phi$. Then $A \subseteq R_{\mathcal{H}}int(c_{\mu}^{*}(A))$. This implies that A is pre regular $\mathcal{H}$-open. Since $(X, \mu, \mathcal{H})$ is a hereditary generalized regular submaximal space, by Proposition 2.4, A is regular $\mathcal{H}$-open. Thus, $A \backslash R_{\mathcal{H}}int(A) = A \backslash A = \phi$. This is a contradiction. Hence $A \backslash R_{\mathcal{H}}int(c_{\mu}^{*}(A)) \neq \phi$.

(b) ⇒ (a)

As by Proposition 2.4, it is sufficient to show that every pre regular-$\mathcal{H}$-open set is regular-$\mathcal{H}$-open. Let A be pre regular-$\mathcal{H}$-open. Then $A \subseteq R_{\mathcal{H}}int(c_{\mu}^{*}(A))$. As a contrary, suppose that A is not regular-$\mathcal{H}$-open. Then $A \nsubseteq R_{\mathcal{H}}int(A)$ and hence $A \backslash R_{\mathcal{H}}int\ (A) \neq \phi$. By (b), $A \backslash R_{\mathcal{H}}int(c_{\mu}^{*}(A)) \neq \phi$. Thus, $A \nsubseteq R_{\mathcal{H}}int(c_{\mu}^{*}(A))$. This is a contradiction. Thus A is regular $\mathcal{H}$ open. Therefore by Proposition 2.4, $(X, \mu, \mathcal{H})$ is hereditary generalized regular submaximal space.

Proposition 2.5

Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then the following statements are equivalent:

(a) $(X, \mu, \mathcal{H})$ is a hereditary generalized regular submaximal space.

(b) $c_{\mu}^{*}(A) - A$ is regular-$\mathcal{H}$-closed for every $A \subset X$.

Proof:

(a) ⇒ (b)

Let $(X, \mu, \mathcal{H})$ be a hereditary generalized regular submaximal space and $A \subset X$. Consider $(X - (c_{\mu}^{*}(A) - A)) = (X - c_{\mu}^{*}(A)) \cup A$. Then $c_{\mu}^{*}(X - (c_{\mu}^{*}(A) - A))) = c_{\mu}^{*}((X - c_{\mu}^{*}(A)) \cup A) \supset (X - c_{\mu}^{*}(A)) \cup c_{\mu}^{*}(A) = X$. Thus $c_{\mu}^{*}(X - (c_{\mu}^{*}(A) - A))) = X$. Hence $X - (c_{\mu}^{*}(A) - A)$ is a dense-$\mathcal{H}$-set. Since $(X, \mu, \mathcal{H})$ is a hereditary generalized regular submaximal space, $X - (c_{\mu}^{*}(A) - A)$ is regular $\mathcal{H}$-open. Therefore $c_{\mu}^{*}(A) - A$ is regular-$\mathcal{H}$-closed for every $A \subseteq X$.

(b) ⇒ (a)

Assume that $c_{\mu}^{*}(A) - A$ is regular $\mathcal{H}$-closed for every $A \subseteq X$. Let A be a dense-$\mathcal{H}$-set in $(X, \mu, \mathcal{H})$. Then $c_{\mu}^{*}(A) = X$. Since $c_{\mu}^{*}(A) - A$ is regular-$\mathcal{H}$-closed for every $A \subseteq X$, $X - A$ is regular-$\mathcal{H}$-closed which implies that A is a regular-$\mathcal{H}$-open set for every $A \subseteq X$. Hence $(X, \mu, \mathcal{H})$ is a hereditary generalized regular submaximal space.

Theorem 2.3

Let $(X, \mu_1, \mathcal{H}_1)$ and $(Y, \mu_2, \mathcal{H}_2)$ be any two hereditary generalized topological spaces. Let $f : (X, \mu_1, \mathcal{H}_1) \to (Y, \mu_2, \mathcal{H}_2)$ be a μ-open surjective function. If $(X, \mu_1, \mathcal{H}_1)$ is hereditary generalized regular submaximal, then $(Y, \mu_2, \mathcal{H}_2)$ is hereditary generalized submaximal.

Proof:

Let $A \subseteq Y$ be a dense-$\mathcal{H}$-set. Since f is surjective, $f^{-1}(A)$ is dense-$\mathcal{H}$-set in $(X, \mu_1, \mathcal{H}_1)$. Since $(X, \mu_1, \mathcal{H}_1)$ is a hereditary generalized regular submaximal space, $f^{-1}(A)$ is regular-$\mathcal{H}$-open in $(X, \mu_1, \mathcal{H}_1)$. Since every regular $\mathcal{H}$-open set is μ-open, $f^{-1}(A)$ is μ_1-open in $(X, \mu_1, \mathcal{H}_1)$. Since f is a μ-open surjective function, $f(f^{-1}(A)) = A$ is μ_2-open in $(Y, \mu_2, \mathcal{H}_2)$. Thus every dense-$\mathcal{H}$-set is μ_2-open in $(Y, \mu_2, \mathcal{H}_2)$. Hence, $(Y, \mu_2, \mathcal{H}_2)$ is hereditary generalized submaximal.

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